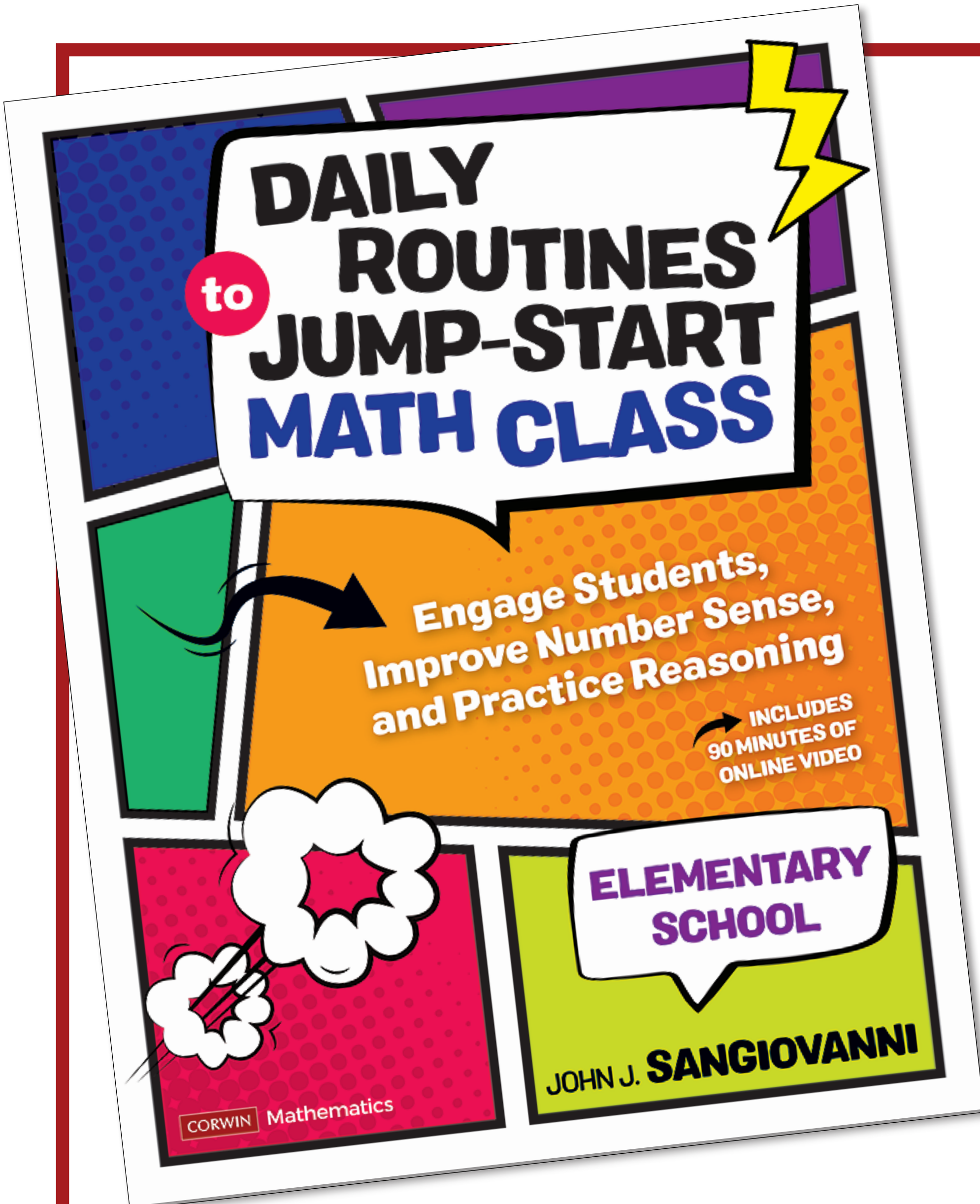


Thank you

FOR YOUR INTEREST IN CORWIN

Please enjoy this complimentary excerpt from *Daily Routines to Jump-Start Math Class, Elementary*, by John J. SanGiovanni. This routine helps students determine the reasonability of their answers and determine efficient methods for estimating.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

ROUTINE 16

MORE OR LESS (ESTIMATING RESULTS)

About the Routine

"Is my answer reasonable?" We want our students to ask themselves this question nearly every time they work with a problem or calculation. Yet, many of our students don't often consider if their answer is reasonable. They engage in mathematics mechanically. They blindly rely on procedures, pencils, or calculators to find solutions. Because of this, they may make computation errors or enter the wrong numbers on a calculator having no idea that their result is impossible or wildly inaccurate. Is 27 + 26 greater than 50? Of course, the numbers could be lined up to use the algorithm. The addends could be decomposed to add tens and ones. Some students might count on from 27 by 26 ones to determine their solution. In some cases, students will reason that 25 + 25 = 50 and because both addends are greater the sum must also be greater. So, is 29 + 19 more than 50? How do we know? How does knowing help? These are the questions at the center of this routine, *More or Less*. In it, students estimate sums, differences, products, and quotients. They compare their estimates to a posted number, which is often a benchmark number. They justify how they estimated the result and how it compares to the given value. This routine is intended to be a mental math activity. So, all desks should be clear of pencils, paper, and other tools for finding exact answers. The routine also plays well when students are called to the carpet in lieu of working at their desks.

More or Less Than 50

29 + 19	27 + 26	9 + 38

Why It Matters

This routine helps students:

- estimate sums, differences, products, and quotients (MP2);
- ask themselves if an answer is reasonable (MP6);
- determine efficient strategies for computing (MP2);
- consider when and if a calculation tool is needed (MP5);
- compare estimates with actual results;
- identify if their computations are accurate (MP6); and
- defend their reasoning and consider the strategies of others (MP3).

What They Should Understand First

Before working with the featured example of this routine, your students should have a firm grasp of the meaning of addition. They should be able to represent addition in a variety of ways and make use of number lines or number charts, which will come in handy when making arguments about their reasoning. For this late first or early second grade example, students should also have learned about adding two-digit numbers, which is built on understanding of place value. In previous lessons, students should have some exposure to prompts that have them look for patterns within expressions or ask them to estimate results before adding. As with all other jump-start routines, students do not need to show mastery of basic facts to work with this routine. Practice and discussion of *More or Less* should have a tangential effect on student fact acquisition.

What to Do

1. Present two or three expressions for students and a number to compare results with. Note that the number of expressions can be limited or expanded relative to student proficiency and time allocations. Also note that it may be best to offer one expression for consideration and discussion at a time.
2. Have students decide how the results of the expression(s) compare to a given value. As noted, this is a mental mathematics opportunity. Students should not use tools to find exact values.
3. Provide time for students to reason about the expression(s).
4. Have students share their ideas with a partner before having a whole class discussion.
5. Begin the whole-class discussion, by identifying who believed that the expression was more or less than the target.
6. Ask students to share how they made their decisions. As students share their thinking, it is important that we probe thinking and ask clarifying questions rather than ask questions to establish how we thought about the expression. Questions might include:
   - How did you estimate your solution?
   - Why did you think about the numbers in those ways?
   - How were those numbers easier or more useful to work with?
   - How did your estimated sum compare to the given value?
   - Might there have been a better or more precise estimate?
   - Could you think about your solution using a different operation?
   - How does your thinking compare to (another student's)?
   - How are the different strategies similar?
   - What is a strategy you might use next time? Why?
7. Honor and explore both accurate and flawed reasoning.
8. Consider providing exact values after discussion so that students can compare and confirm their more or less estimates.

Anticipated Strategies for This Example

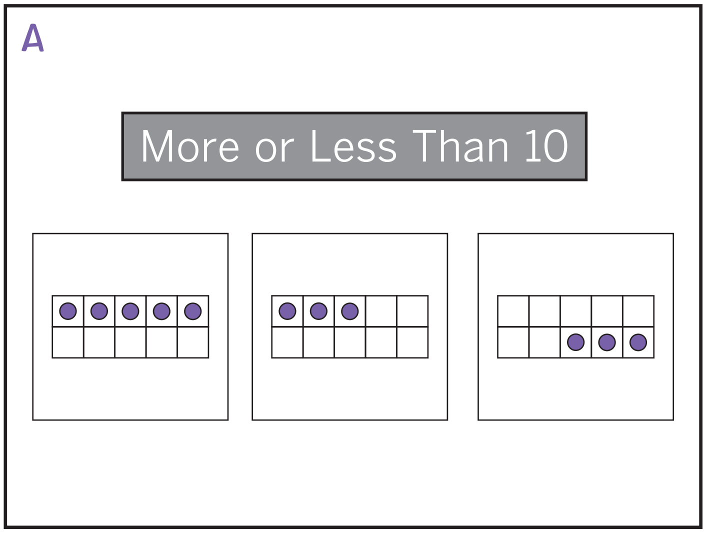
More or Less Than 50

29 + 19	27 + 26	9 + 38

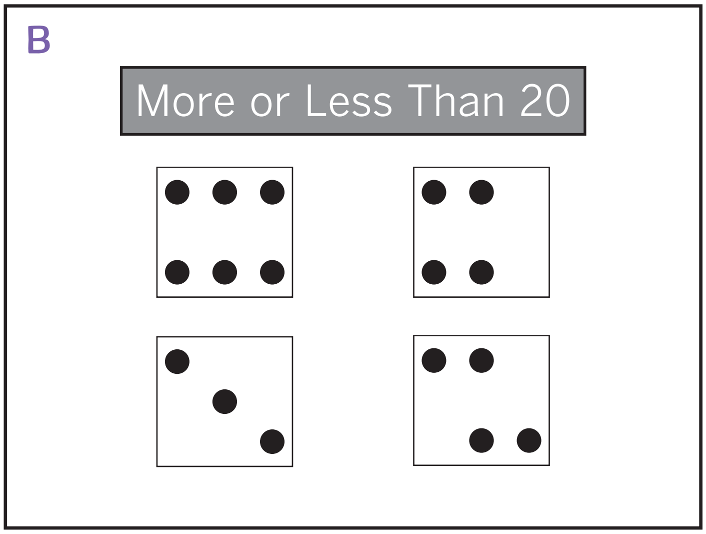
It is entirely possible, if not somewhat likely, that some students will attempt to use a procedure to find the sums and then compare them to 50 (prompted comparison in the featured example). This may occur even if algorithms haven't been taught in school. Some students might describe thinking about base 10 blocks in their heads or using a mental number line. While possible, this imagining of physical representations and number lines may prove to be problematic as numbers become more complex. From time to time, students offer this *strategy* as another idea when it is not necessarily something they even did. You should acknowledge the strategy but also be careful not to inadvertently favor it more or overemphasize it. Strategies of partial sums, adjusting, and compensation might be offered. You should also listen for comparison strategies grounded in reasoning about the size of the numbers and the operation. For example, 9 + 38 must be less than 50 because 10 + 40 is 50 and both addends (9 and 38) are less than 10 and 40 respectively. From time to time, you should inject these ideas into conversations when students don't offer them.

MORE OR LESS—ADDITIONAL EXAMPLES

A. Understanding for this routine is rooted in activities and opportunities in kindergarten and early first grade. Students at these levels can reason about the results of addition and subtraction, too. You can provide students with physical or visual representations to help them make sense of the quantities being added and as well as their comparisons. Example A shows what *More or Less* might look like for these students. Here, three 10 frames are posed in order to compare the sum of these frames with 10. Earlier experiences might first begin with two 10 frames. Even so, you should provide opportunties for three 10 frames at some point. In this example, students might discuss giving three to five and seeing that only two more empty spaces are left. Others might put the two threes together noting that it makes more than half of the 10 frame.



B. Proficiency with making and decomposing friendly numbers and benchmarks is another lynchpin for student success. As with example A, you can expose students to these ideas and comparisons in the earliest grades. Ten frames are one way to do so. Dominos or dot cards, as shown in example B, are other options. Now, students are presented with four cards. This is intentional in order to elicit ideas that 10 and 10 make 20 and that only one row (6 and 4) makes 10. Some students might only be able to see the cards vertically at first. Thus, your discussion afterwards will be critical for helping them see other strategies or possibilities. As with Example A, students in Example B might rely on counting all of the dots or pips. As this happens, it is critical that you highlight and reinforce more efficient strategies offered by other students. You should acknowledge students who count by ones and ask them to think about how their results compare to other grouping and counting strategies.



C. *More or Less* works with any operation and any size or type of number. You likely won't offer all of the expressions in Example C all at once to students in any one grade. Instead, the example is used to show what different options might look like in different grades. The first prompt, 7×5 is an example of how students might think about basic facts. The second prompt, $712 - 644$, shows how three-digit subtraction might work or subtraction in general. Here, students have to consider if the difference will be more or less than 25. Students might try to regroup, some might count back, some might count up, and others might think about the space between 650 and 700 being greater than 25 itself.

C

More or Less Than 25

7×5	$712 - 644$	$56 \div 2$

D. Similar to Example C, the expressions in Example D are provided to give a sense of the range of the routine. However, Example D is different in another way. Its *comparison number* (81) is not a friendly number or benchmark. These numbers are good to include as comparisons in the routine. Keep in mind they you might reserve them for students who have experienced the routine a bit. Numbers like 81, 276, and so on can be much harder to think about. First, working with friendly, benchmark numbers helps students build a foundation that they can then apply to these more *complicated* numbers. The strategies students use are likely to remain similar. For example, the first prompt ($163 - 75$) might be thought of as a combined difference of 25 (100 and 75) and 63 (100 and 163), which is more than 81. In the second, students might quickly reason that $3 \times 30 = 90$ so 3×34 must be more than 81. And in the last, students might think that $600 \div 6 = 100$ and 592 is quite close to 600 so then the quotient must be close to 100.

D

More or Less Than 81

$163 - 75$	3×34	$592 \div 6$

MORE OR LESS VARIATION—FRACTIONS

Student challenges with operations and sense making often increase when fractions are included in problems and computations. In these situations, reasoning about addends or estimating sums and differences is even more important. *More or Less* can be adjusted so that students can transfer their ideas and strategies to these *new* numbers.

E. Early use of *More or Less* should simply ask students to compare the results of fractions to 1. In time, you can use multiple addends so that students confront comparisons of larger whole numbers such as 5 or 6. Using this routine with fractions, also helps students revisit ideas such as relating fractions to benchmarks such as 0, $\frac{1}{2}$, and 1. This skill is a helpful for comparing fractions. It is also quite helpful for estimating results of adding or subtracting fractions. In Example E, students are to consider if $\frac{4}{10}+\frac{3}{4}$ is more or less than 1. Reasoning that $\frac{4}{10}$ is almost half and that $\frac{3}{4}$ is a good bit more than $\frac{1}{2}$ enables them to conclude that the sum is more than one. This determination is most useful when exact sums are needed and they must consider if their results are reasonable. The right example is a glimpse at how subtraction with fractions might play out in the routine.

E

More or Less Than 1

$\frac{5}{8}+\frac{4}{8}$	$\frac{4}{10}+\frac{3}{4}$	$1\frac{3}{5}-\frac{3}{8}$

F. Example G suggests that students should first work with comparing results to 1. But in time, you can begin to pose more interesting expressions and comparisons. Yet again, the strategies are likely to be similar. Students might consider how close each addend is to 0, $\frac{1}{2}$, or 1 and possibly 2 (as in the right prompt). In the left prompt, common denominators and exact calculations might be considered. Strategies that rely on exact solutions is fine. In some cases, students can find exacts more efficiently especially if all terms have common denominators. But common denominators are not always used. When denominators aren't common, as with the left prompt, students might think about the sum of $\frac{1}{8}$ and $\frac{2}{4}$ being less than 1 and so the other addend must be at least a little more than 2, which it is not.

F

More or Less Than 3

$\frac{3}{2}+\frac{1}{8}+\frac{2}{4}$	$\frac{1}{6}+\frac{2}{3}+\frac{16}{8}$

MORE OR LESS VARIATION—UNKNOWNS

Your first reaction to Examples G and H is probably "Our kids don't have to do that!" But, can they? Elementary students are expected to write equations to solve problems with unknowns in any position. Using equations in this routine provides exposure and discussion to help students develop comfort and confidence with equations. It also helps them make sense of how numbers are related to one another in an equation—which is something they have to do.

G. Each of the *More or Less* prompts in Example G could easily represent an elementary word problem. As we know, student success with word problems can be disappointing. Often, we wonder if our students even considered if their answer made sense. Sometimes that consideration is in relation to the context of the problem or the question that it is asking. Other times, it is in relation to the results of their computations. What is you strip away everything but the equation and ask them to simply think about the numbers in the problem? The result is Example G. Here, it's likely that many elementary students will quickly recognize that the value of the unknown in the first prompt must be much more than 25. In fact, the others are quite obvious, too. For example, if they know that there are four 25s in 100, the unknown in the middle equation must be more than 25.

G

More or Less Than 25

? + 7 = 49	150 ÷ 4 = ?	147 = 201 − ?

NOTES

H. You can apply fractions and unknowns in equations to *More or Less*. You can easily adjust it to use with decimals and even unknowns in equations that have decimal numbers. Example H shows how decimals might be incorporated into the routine. This adaptation of the routine is better for students in middle to late fifth grade or students who need enrichment and/or extension opportunities. As with other unknowns, students do not have to apply procedure to solving the equations. Substitution and reasoning will suffice. The first equation is one that students often get wrong when applying procedure rather than reasoning. This happens as the numbers stoke a basic fact memory (9 + 8 = 17). But when you ask them to think about the unknown being more or less than 8, they are forced to pause and consider how adding $\frac{7}{10}$ to a whole number affects the sum (it will have $\frac{7}{10}$). In the last example, students who recognize that 8 ÷ 1 = 8 will conclude that 8.4 ÷ 0.4 must be greater because 0.4 is smaller than 1 meaning more will fit in 1, 8, or ultimately 8.4. The numbers used in this equation might provoke students to find an exact quotient (2.1). The debate will be which is more efficient and does efficiency change based on the numbers in the equation.

H

More or Less Than 8

1.7 + ? = 9.1	13.1 – 5.6 = ?	8.4 ÷ 0.4 = ?