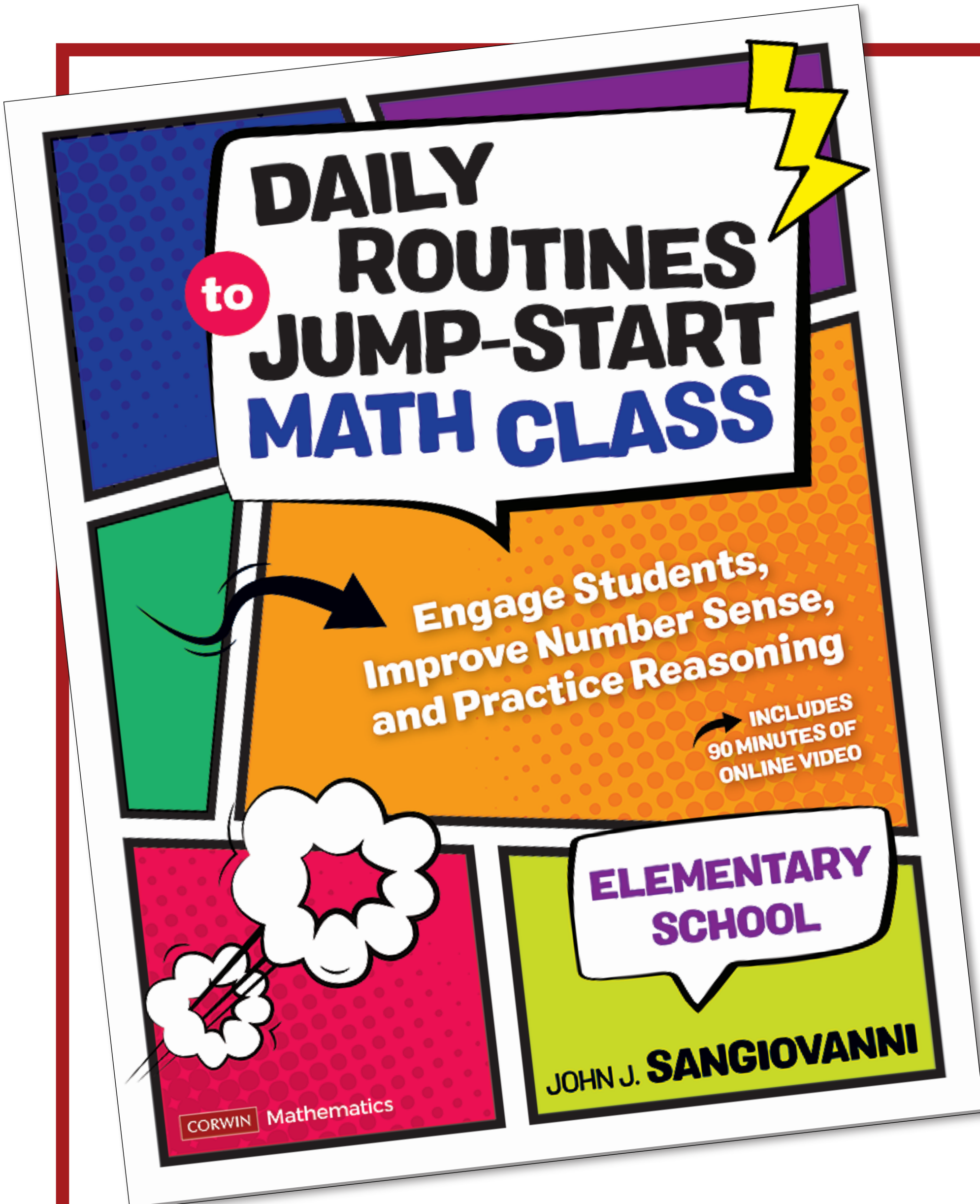


Thank you

FOR YOUR INTEREST IN CORWIN

Please enjoy this complimentary excerpt from *Daily Routines to Jump-Start Math Class, Elementary,* by John J. SanGiovanni. This routine helps students develop a more robust understanding of—and flexible thinking about—numbers and their relationships, which positions them for greater success when working with numbers.

LEARN MORE about this title, including Features, Table of Contents, and Reviews.

ROUTINE 13

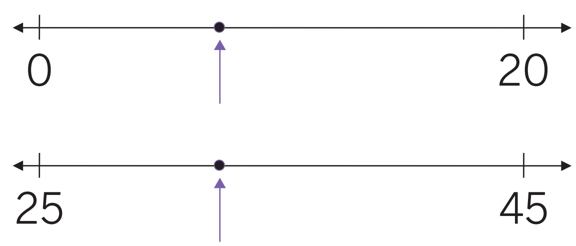
WHERE'S THE POINT? (NUMBER LINE RELATIONSHIPS)

About the Routine

Understanding how numbers are related helps students compare, estimate, and calculate. Seeing relationships between numbers helps them recognize how dynamic number relationships are and helps us make sense of the magnitude of number. Number lines are a useful tool for seeing the relationship between numbers. However, number lines often feature fixed endpoints or specific tick marks that may lead students to think about numbers in limited ways. Such rigidness can inhibit your students' ability to reason about the size of or relationship between numbers. This routine helps students develop a more robust understanding of—and flexible thinking about—numbers and their relationships, which positions them for greater success when working with numbers. This routine presents locations on number lines with limited information about the values those locations might represent. Students then reason about how they can determine what the value might be. They are likely to consider midpoints, quarter-points, and other benchmarks to identify the value. In some situations, like the one presented here, they may also reason about how the endpoints in a number line change in the same way. The difference between endpoints in this example is 25 so the midpoint must be 25 greater than the first midpoint. Students then share their solutions and reasoning with partners before taking part in a class discussion. At that time, you facilitate a conversation about strategies and reasonable, if not exact, possibilities for the location on the number line.

Where's the Point?

- What number is the arrow pointing to?
- How do you know?



Why It Matters

This routine helps students:

- reason about number relationships using number line models (MP4);
- develop their ideas about half of a number and relationships to other benchmarks (MP2);
- reason about the *closeness* of values. For example, 47 and 54 may be different locations on a number line but they are both *close to* 50 on a number line (MP2);

- build number sense and confidence for working with numbers;
- think flexibly about quantities and models; and
- defend their reasoning and consider the approaches of others (MP3).

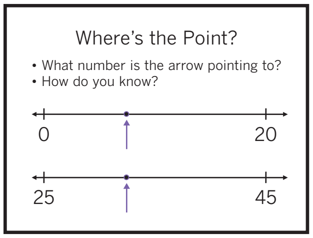
What They Should Understand Before the Routine

In this example of *Where's the Point?,* students find values by reasoning about the relationship between two whole numbers. The first number line has a traditional left endpoint whereas the other endpoints may be quite unusual or unfamiliar. Your students should be aware that endpoints on number lines are not static and that any value rather than the *standard* 0 and 100 or 0 and 1,000 can be used for an endpoint. This routine is intended to refine this understanding as it may not be fully developed. Your students should understand how a number line works and have had some experience with open number lines before working with the routine. They should be able to add and subtract friendly numbers. It will also be helpful if your students are able to reason about numbers that are half of other numbers although they might not be able to find halves fluently and/or with precision. These number lines do not have specific, ticked intervals. Because of this, this routine might be best saved for students in late first grade or early second grade.

What to Do

1. Determine the number of number lines to use in the routine. Note that not all number lines must be discussed during the routine due to time restrictions. Two or three number lines will be sufficient.
2. Present students with one or more number lines with specific endpoints and an arrow pointing to a location on the number line, as in the example provided. Note that it may be desirable to investigate one number line without the other number lines appearing before moving to a second and potentially a third number line.
3. Prompt students to think independently about the relationship between the numbers to determine the value of the unknown location.
4. After finding a value, ask students to share their solutions and reasoning with a partner.
5. Bring students together. Solicit and record various possibilities offered by students.
6. After collecting different possibilities, explore reasoning that justifies those solutions.
7. Have students share their reasoning and ask others to signal if they used a similar strategy.
8. Ask students to share their thinking and probe ideas about how they determined the value of the missing point. Questions to ask might include:
   - How did you find the value of the arrow?
   - What was half of the distance between these values?
   - How did half help you think about the possible solution?
   - Were there other relationships that were helpful?
   - Did anyone think about the number line in a different way?
   - How did you find the value of the second number line?
   - Did anyone begin with the bottom number line before working with the top number line? How was that helpful?
9. Honor and explore both accurate and flawed reasoning.
10. Consider providing students with calculators or 100 charts to help them accurately find values.
11. If time permits, consider extending the routine by creating a new number line with a similar location and a new set of endpoints or ask students to do this.

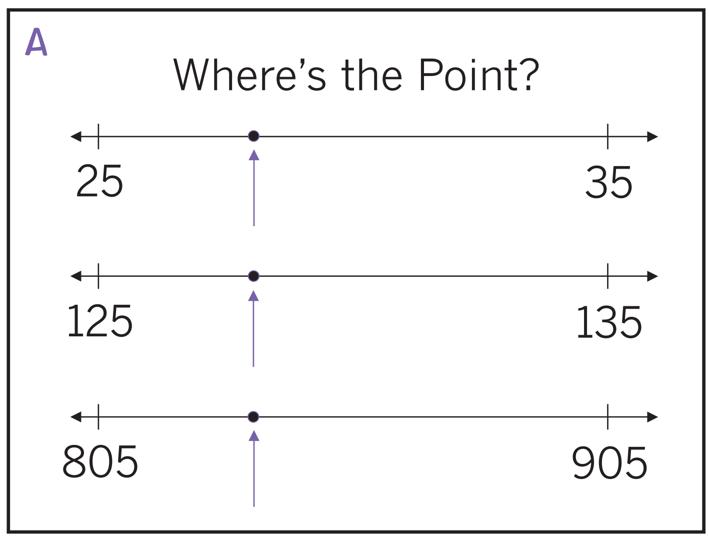
Anticipated Strategies for This Example



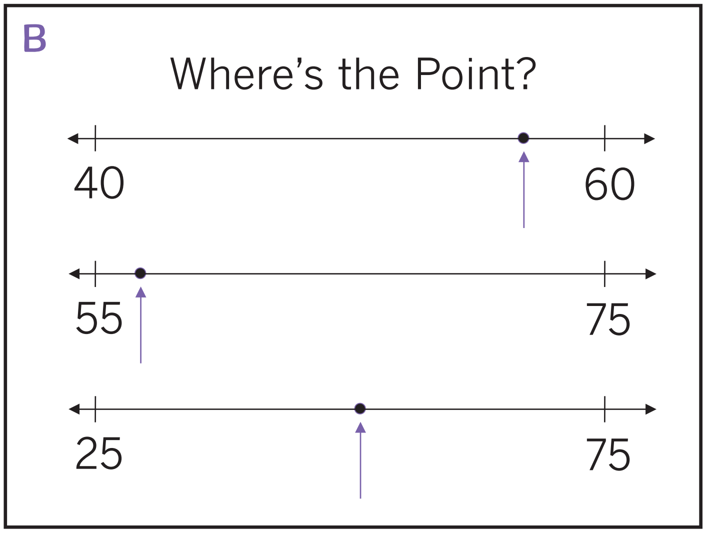
Be prepared for some of your students to identify the value of the arrow as the next counting number from the last known value. In this example, they would share 1 for the top number line and 26 for the second number line. This might hint at students who aren't quite ready for the routine yet. It might also signal that they are not yet thinking about how numbers are related spatially. They might not be thinking about how far from 0 is 20, 25, 50 or 100. You can strengthen their understanding by examining spatial relationships between numbers on number charts. You can also help them make connections between number charts and number lines during discussion. Students might use another strategy by picking a *halfway* number each time that an arrow is between two numbers regardless how close the arrow is to the middle of those numbers. Students might also choose numbers at random without any reasoning or justification about their selections. This should improve as experience with the routine is gained. It may be helpful to give landmarks on the number line to improve student accuracy with the routine. Here, you might offer where 10 and 35 on each number line or where 5 and 30 or 15 and 40.

WHERE'S THE POINT?—ADDITIONAL EXAMPLES

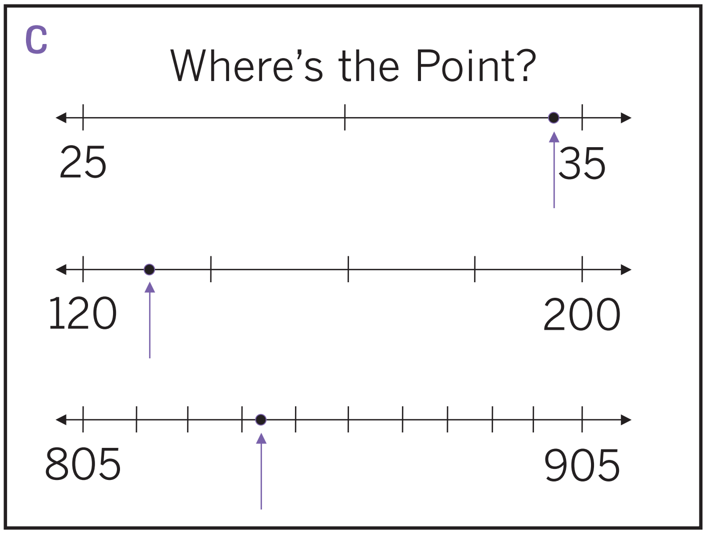
A. *Where's the Point?* can take advantage of different relationships on a number line. Two adjacent number lines can focus students on similar changes to the endpoints of a number line. In Example A, the top and middle number lines have endpoints that have changed by 200. Therefore, the value of the arrow should change by 200. The middle and bottom number lines play on the concept of the interval between endpoints. In this same example, the endpoints on each number line are 110 apart. The arrow is the same distance from each endpoint on both number lines so the value might be 20 or so more than the left endpoint. Keep in mind that these three number lines are shown together for convenience. The two distinct ideas (same change in the top/middle number lines and same interval in the middle/bottom number lines) that they represent would be best reserved for separate days.



B. As your students' reasoning about relationships grows, you can begin to shift the placement of the unknowns. You may continue to highlight benchmark locations such as midpoints as shown in the last number line in Example B. The location can also be shifted more dramatically. The top number line shows an interval of 20. It might be especially challenging for students to think of numbers so close to 50 (40 and 60) as also being far apart. Students might determine that the marked location on the top number line is 58. Others might suggest 57 or 56. You should accept reasonable answers. Strategies should be discussed and explored. In this example, students are likely to find some way to quantify the space in between. Some may think about 50 as the midpoint and then partition the remaining distance in some way. Others may attempt to partition the entire space between 40 and 60.

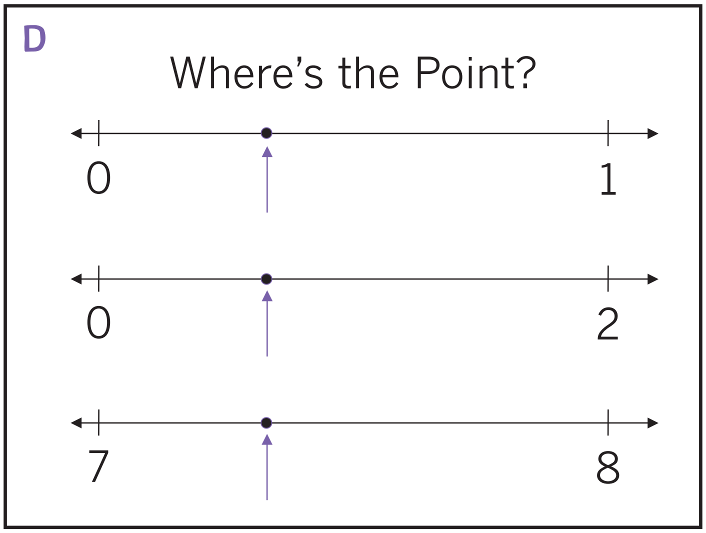


C. For some students, visual references to benchmark locations may be needed to support their reasoning. This version might actually be the place to begin for many students. It provides tick marks to help them make sense of the distance between numbers. These tick marks can support counting intervals as well as hold the location of certain numbers for students as they determine the possible value of the noted point. The number of and interval of tick marks should be based on the needs of students. In time, tick marks should be lessened and removed altogether.



It might be wise to begin with traditional endpoints on number lines be they 0 and 100, 0 and 1,000, or something similar. You can nurture better developed strategies that can be applied to more unusual number lines by beginning with these endpoints. You can also help students see relationships more clearly by using related number lines in the same routine. For example, one number line can have endpoints of 0 and 100 and the other can have related endpoints such as 200 and 300. Example C shows a different strategy for supporting students.

D. Later in elementary school, *Where's the Point*? can be modified to bolster student understanding of fractions. When working with fractions, students often experience similarly stagnate endpoints of 0 and 1. This may be a necessary starting point. However, your students should be moved to reason about other possibilities as well. In Example D, students first think about a location between 0 and 1. They might find it to be $\frac{1}{3}$ or $\frac{1}{4}$. The next two number lines show a targeted value in the exact same location as the first number line. Yet, these number lines have different endpoints. The endpoints of the middle number line are one more than those of the top therefore the arrow is one more or $1\frac{1}{3}$ or $1\frac{1}{4}$. The endpoints of the bottom number line can be thought of in a similar way creating a similar justification for the arrow's value to be $7\frac{1}{3}$ or $7\frac{1}{4}$. This version of *Where's the Point?* is a good opportunity for you to help students see how fractions are related and that the distance between any two consecutive whole numbers can be thought of in similar ways.

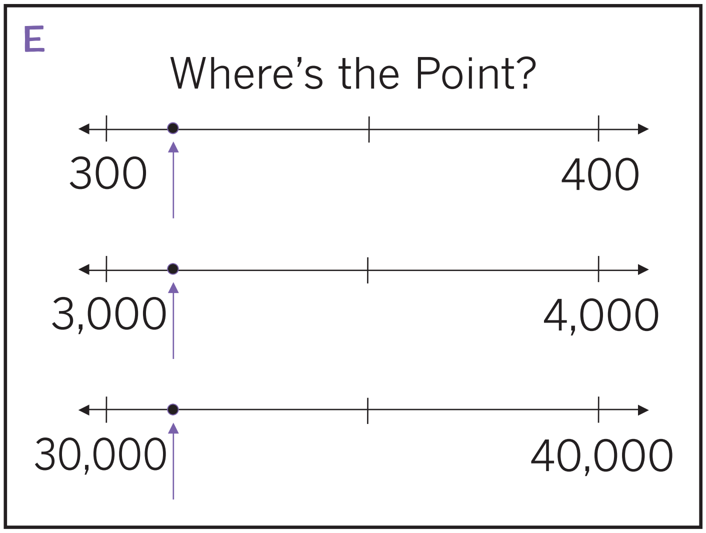


NOTES

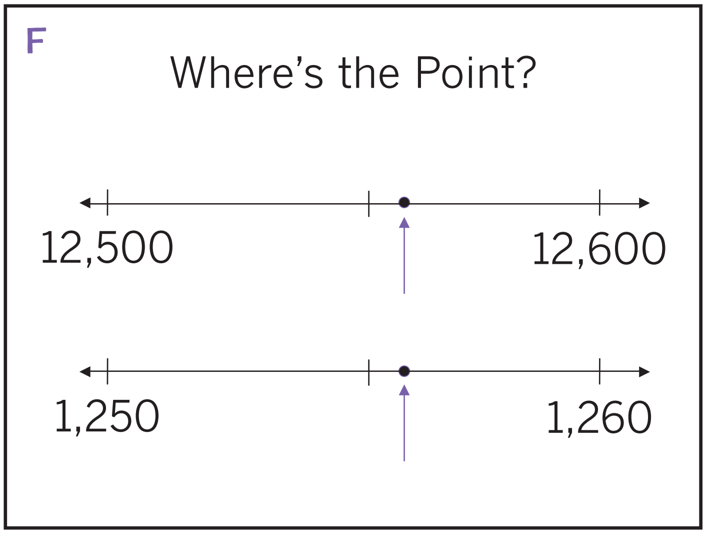
WHERE'S THE POINT?
VARIATION—BASE 10 RELATIONSHIPS

Numbers and number relationships can become more and more challenging to think about as the number of place values increase. Essentially, it can be hard to think about very big and very small numbers (Krasa & Shunkwiler, 2009). Students learn about powers of 10 in later grades but the understanding of the concept may not be fully realized at first. *Where's the Point?* can help your students see relationships between *similar* numbers sooner and in turn serve as a foundation for work with powers of 10.

E. Example E shows how powers of 10 can be applied to *Where's the Point?* Here, the first number line shows a distance of 100. Many fourth and fifth graders will be able to easily determine the location to be about 310 or 320. The next number line has endpoints that are 10 times greater than the first number line. In fact, 300 and 400 would not even appear on this second number line. The location of the point has not moved so that point is 10 times more than the one just above it. 3,100 or 3,200 would be reasonable. You might discuss the midpoints of each number line as well. In fact, these might be the starting places for students' reasoning rather than thinking about 10 times more. The bottom number line is 10 times more than the middle. The value of the point noted by the arrow can be found in a similar way. It can be thought of as being 31,000 or 32,000. The potential of the routine is clear in this instance. Yet, you might start this example with endpoints of 30 and 40 before moving to larger numbers to better support your students.



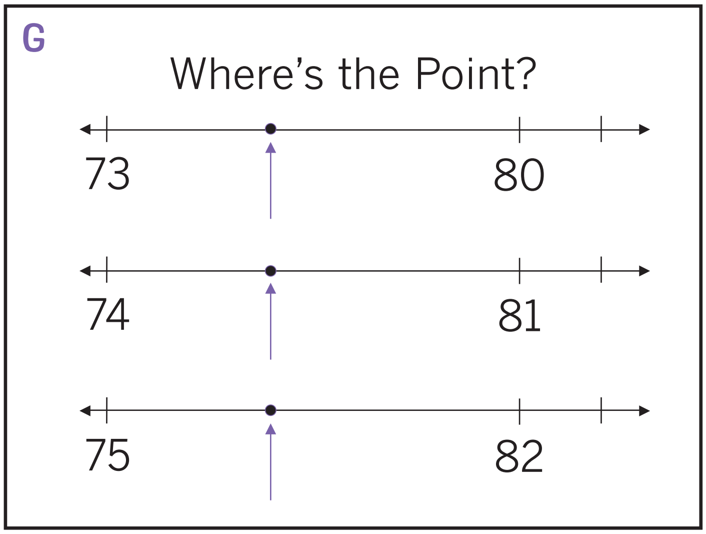
F. Example F builds on the idea of powers of 10. However, it reverses how the numbers change. Finding a tenth of a number might be quite the challenge for students. You can adjust the routine so that all number lines are provided at the same time.



WHERE'S THE POINT? VARIATION—SHIFTING POINTS

Where's the Point? can build other ideas about number and number relationships through careful manipulation of the number lines and endpoints.

G. Points on a number line change in various ways. Two seemingly identical locations will have different values due to other known points. This routine has made use of endpoints but other locations can be just as useful. These manipulations can be fodder for some enthusiastic debates about the value of unknown locations. Example G shows how you can shift the points on a number line and how endpoints can be marked with unknown values. You might withhold these variations of *Where's the Point?* until after your students have had quite a bit of time working with and reasoning about values on empty number lines.



H. Example H offers one last variation for the routine. Here, the right endpoints in each number line are the same as is the position of the arrow on each number line. If Example G didn't ignite debate Example H is sure to do so. Your students might attempt to simply adjust each value by adding 50 to each new number line. In fact, many students will insist that *is* how one must find the unknown values. Others might argue that the space between the endpoints is being *squished* resulting in a changed value of the arrow but not a change of 50. Arguments from both sides will offer great insight into student thinking.