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Foreword by
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THE

5 PRACTICES

in Practice

Successfully Orchestrating Mathematics Discussions in
Your Middle School Classroom

Thank you

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Please enjoy this complimentary excerpt from *The Five Practices in Practice, Successfully Orchestrating Mathematics Discussions in Your Middle School Classroom* by Margaret (Peg) Smith and Miriam Gamoran Sherin. The following excerpt demonstrates strategies to anticipate student responses in problem solving, including planning to respond to students using assessing and advancing questions, and preparing to notice key aspects of students' thinking in the midst of instruction.

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CORWIN Mathematics

Unpacking the Practice: Anticipating Student Responses in Problem Solving

What is involved in anticipating students' responses? This practice involves getting inside the problem (thinking about how you and others might solve the task), planning to respond to students using assessing and advancing questions, and preparing to notice key aspects of students' thinking in the midst of instruction. The figure below highlights the key components of this practice.

Figure 3.1 • Key questions that support the process of anticipating students' responses

| WHAT IT TAKES	KEY QUESTIONS
Getting inside the problem	How do you solve the task?
	How might students approach the task?
	What challenges might students face as they solve the task?
Planning to respond to student thinking	What assessing questions will you ask to draw out student thinking?
	What advancing questions will help you move student thinking forward?
Planning to notice student thinking	What strategies do you want to be on the lookout for as students work on the task?

Getting Inside the Problem

The first step is to get inside the problem! Many teachers find it useful to start by thinking about their own approach. How do you solve the task? You will want to think generally about the approach you use and at a detailed level about steps in your process (which may be different from someone else). Next consider how others might approach the task. You might investigate the problem using a different representation or think about how manipulatives might shape the way students explore the task. Do some approaches move students more easily toward the learning goals you established? You could also think about whether the task has different entry points. Often when students begin a task by working on different parts of the problem, their solutions look different (Lambert & Stylianou, 2013). Finally, as you explore these various approaches, keep in mind any challenges you think students will face as they solve the task. Are certain parts of the task likely to be difficult for students? Do you expect that students who use certain approaches will face particular kinds of challenges? Where do you think students might get stuck?

The State Fair

You are going to the Kentucky State Fair in August. You are trying to figure out how much you should plan to spend. The graph below shows how much three different people spent after going through the main gate and then buying their ride tickets. Every ride ticket is the same price.

Money spent at the Kentucky State Fair
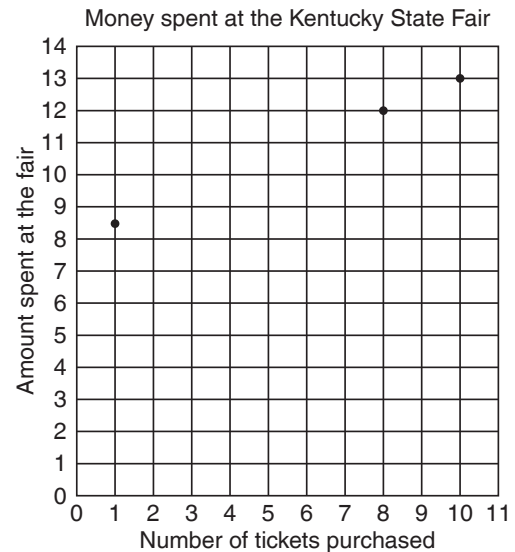
1. After entering the fair, you decide to buy four ride tickets. What will be your total cost for attending the fair? How do you know?

2. Describe how the cost increases as you buy more tickets. Be specific.

3. After entering the fair, you decide you want to go on a lot of rides. What will be the total cost for attending the fair and then purchasing 15 ride tickets?

4. Write a description, in words or numbers and symbols, that can be used to find the total cost after entering the fair and purchasing any number of tickets.

5. How does the ticket price appear in your description or expressions?

6. How does the ticket price appear in the graph?

Extension

1. If you went to the Kentucky State Fair, how many ride tickets could you buy with $25.00?

2. If you could enter the Kentucky State Fair for free, how would the graph look different?

Source: Jennifer Mossotti. Ferris wheel photo by Hannah Morgan on Unsplash.

Analyzing the Work of Teaching 3.1

Getting Inside a Problem

Solve the State Fair task in at least two different ways. Then consider:

- What did you need to know to solve the task?

- What do you think might be challenging for students about this task?

Getting Inside a Problem—Analysis

While there are several ways you might approach this problem, we will explore two possible methods here: connecting the points and finding the slope. In connecting the three given points, you would note that they all fall on the same line, and that the line intersects the y-axis at (0, $8.00). This could lead you to conclude that it costs $8.00 to enter the fair. By comparing the two points (0, $8) and (1, $8.50) you can see that it costs 50¢ per ticket and that the equation for the line contain these four points would be $c = .50t + \$8.00$. You can then use the equation to find the cost of 4 tickets.

Alternatively you could take any two of the three given points and determine the slope of the line that contains these two points by using the slope formula, $m = \dfrac{(y_2 - y_1)}{(x_2 - x_1)}$. For example, if you use (1, $8.50) and (8, $12.00) you would get $\dfrac{(12 - 8.50)}{(8 - 1)}$, so m would be .50 or 50¢. You could then use the slope-intercept form of a line ($y = mx + b$) and substitute .50 for m and one of the three points for x and y in order to find b. This would result in the equation $y = .50x + 8$. You can then use the equation to find the cost of 4 tickets.

In order to use the first strategy (connecting the points), you would need to know that if all the points lay on a line you can represent the line with the equation $y = mx + b$; that a line can be extended in either direction and the value of the point where the line intersects the y-axis is of the form (0, y); that if the difference in the x-values is one the difference in the y-values is the rate of change; and that m is the rate of change and b in the y-value of the y intercept in the equation $y = mx + b$. In order to use the second strategy (finding the slope) you would need to know the points all fall on the same line; the formula for slope; and the slope-intercept form of a line. In both of these solutions, you need to know a fair amount of algebra!

Middle school students may be challenged by this task because they don't yet have a solid foundation in algebra that would give them access to the same methods we might gravitate to. For example, they may not realize that the y-intercept can be something other than (0, 0) and that it has meaning in this context. They may not recognize that there is a constant rate of change since the points given are not consecutive. They also may not realize that the three given points do not represent all the options for buying tickets.

Through collaboration with colleagues, Mrs. Mossotti identified several possible methods for solving the task (see Figure below) that went beyond how she might have solved the task herself. Based on using a similar task last year, she expected to see some students create a table using the three points from the graph (1, $8.50), (8, $12.00) and (10, $13.00) without knowing what to do next (Solution A). Mrs. Mossotti suspected that the idea of the entrance fee could be "an issue for a lot of students." As she explained, "I think when they look at, for example, 8 and 12, they might think that 12 is the cost for 8 tickets without actually reading the y-axis to realize that it's the amount that they've spent total." In fact, she mentioned that some students might try to divide the cost by the number of tickets to find the cost per ticket, noting "this would not be correct at all, because they're going to find three different rates for tickets" (Solution B).

Mrs. Mossotti also described a few other ways that students might determine the price per ticket. In one approach, students would recognize that if 8 tickets cost $12.00 and 10 tickets cost $13.00 that two tickets must cost $1.00 (Solution C). Mrs. Mossotti also suggested that, "some of them are going to use the fact that the graph looks like a line and connect the points and then go from there to start to figure out the price for each ticket." She anticipated that the y-intercept at (0, $8.00) would be particularly notable for some students, suggesting to them that the cost of 0 tickets was $8.00, and that 1 ticket was $0.50 (Solution D). Mrs. Mossotti predicted that other students might use the information given to determine the total cost for a different number of tickets, noticing the relationship between the three given points (Solution E). Throughout this process, Mrs. Mossotti and her colleagues considered in detail the different ways her students might approach the State Fair task and also the reasoning underlying the students' ideas.

Figure 3.2 • Anticipated solutions to the State Fair Task generated by Mrs. Mossotti and her colleagues

A. Make a table with values from the graph

Number of Tickets	Total Spent
1	$8.50
8	$12.00
10	$13.00

Student creates a table using the information about the three points on the graph.

B. Determine different unit rates for tickets prices

$$\frac{8.50}{1} = 8.50 \qquad \frac{12}{8} = 1.5 \qquad \frac{13}{10} = 1.3$$

Student divides the total spent by the ticket quantity for each point on the graph and comes up with three different "unit rates."

C. Determine the price per two tickets

Student uses the points (8, 12) and (10, 13) to determine that two tickets have a cost of $1.00.

E. Determine price per ticket at 50¢

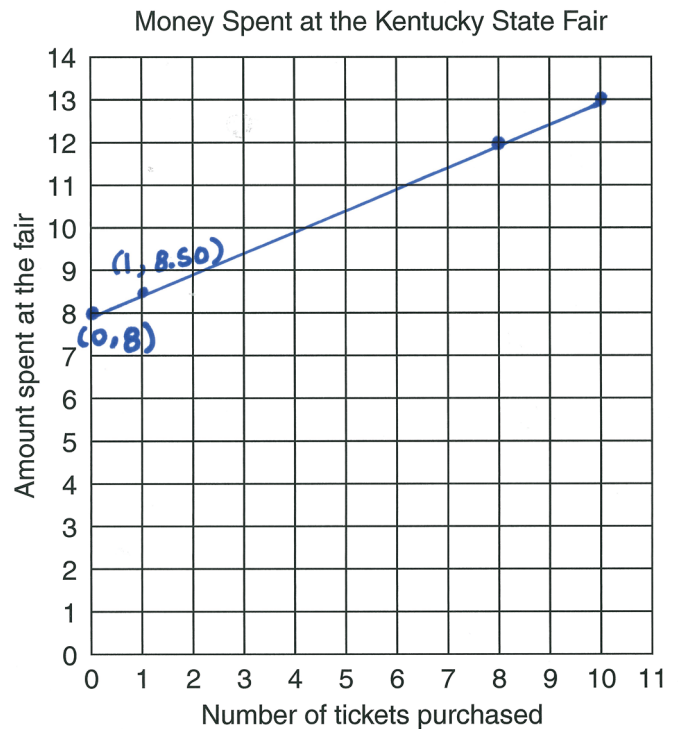| Number of Tickets	Total Spent
0	$8.00
1	$8.50
2	$9.00
3	$9.50
4	$10.00
5	$10.50
6	$11.00
7	$11.50
8	$12.00
9	$12.50
10	$13.00

Student determines that the total amount spent is calculated by taking $8.00 and then repeatedly adding 50¢ depending on the number of tickets purchased.

D. Connect three points with a line to determine entry fee and ticket price.

Money Spent at the Kentucky State Fair

Student connects the three points on the graph with a line and sees that at the y-axis the value is $8.00 and determines that it must cost $8.00 to enter the fair without buying any tickets. Since it costs $8.50 for 1 ticket, this means it must cost 50¢ per ticket. Student sees that the line rises half a unit on the y-axis for every 1 unit on the x-axis.