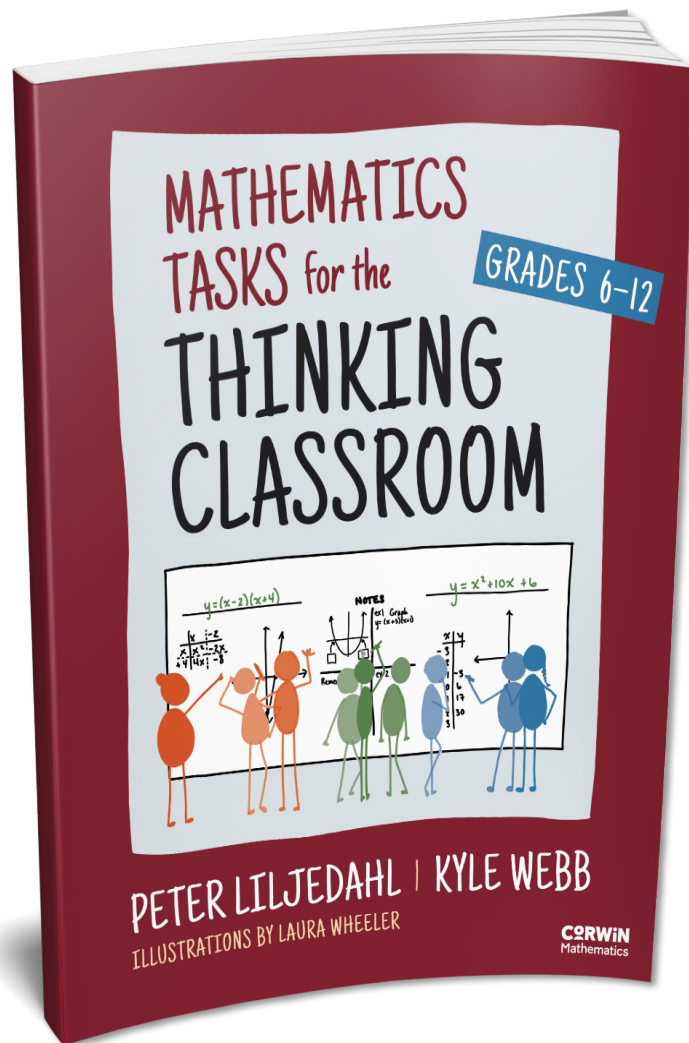




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Mathematics Tasks for the Thinking Classroom, Grades
6-12.

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TASK 18: BROKEN GLASS

TASK

Given only two drinking glasses, determine the highest floor you can drop a drinking glass from without it breaking. Come up with a strategy that, in its worst case, will require the least number of drops.

Grades: 8–12

Content Potential: arithmetic series, modeling

Perseverance Scale:

Grade Level	8	9	10	11	12
Perseverance Level	3	3	3	2	2

Macro-moves: give tasks verbally, mobilize knowledge

Competencies: guess-and-check, willingness to take risks

LAUNCH SCRIPT

Teacher: Hello, mathematicians. I have a job for us. I was contacted by a company that makes outdoor dishes. They have started to manufacture decorative drinking glasses, and they need to know how strong they are so that they know what to tell their clients who live in high rise buildings. They need us to figure out what is the highest floor that a glass can be dropped from and not break.

Teacher: To help with this they have given us two drinking glasses and use of a 100-floor apartment building. Come up with a strategy that will minimize the number of drops needed.

CREATING ACCESS

- For students who are having difficulty starting, tell them that it is a 5-floor apartment building. Then 10 floors, 25 floors, and so on.

EXTENSION SCRIPTS

- A. What if it was a 200-floor apartment building?

AUTHOR SOLUTION(S)

This task falls into a category of tasks where you are looking for the best worst-case scenario. For example, you can drop the first glass from floor 99. If it does not break, then go to floor 100. If it breaks, you now know that 99 is the answer. This is a best-case scenario. But what if it did break at floor 99. Now you only have one glass left and you still need to provide an answer. So, you drop it from floor 1. If it does not break, you need to go to floor 2, and so on, all the way up to floor 98. So, although 2 drops was your best-case scenario for this strategy, the worst-case scenario is where the second glass either breaks or survives a drop from floor 98. That would require a total of 99 drops. The worst-case scenario for this strategy is 98. We need to find the best worst-case scenario.

So, let's start with a strategy where we drop the first glass from floor 50. If it does not break, then we go to floor 100. That is, we go up by increments of 50. The worst-case scenario would be if it survived on 50, broke on 100 (2 drops so far), and then we have to try floor 50, 51, 52, . . . , 99 (49 more drops). This is a total of 51 drops.

Let's try increments of 20. The worst-case scenario is if the first glass survives floors 20, 40, 60, 80 and breaks on 100 (5 drops so far). Then we need to use the second glass to test 81, 82, 83, . . . , 99 (19 more drops) for a total of 24 drops. This is a better worst-case scenario.

Now let's try increments of 15. The worst case is that the first glass survives floors 15, 30, 45, . . . , 75 and breaks on 90 (6 drops so far). Then we need to use the second glass to test 76, 77, 78, . . . , 89 (14 more drops) for a total of 20 drops. This is an even better worst-case scenario. Note that for an interval of 15, the first glass surviving floor 90 and breaking at floor 100 is not the worst-case scenario as it would only require 16 drops (7 drops for the first glass and 9 for the second). We can keep going like this using different intervals and we see that there are lots of different intervals that have 19 as the worst-case scenario.

Interval	Floor 1 st glass breaks	Drops for 1 st glass	Drops for 2 nd glass	Total drops
50	100	2	49	51
20	100	5	19	24
15	90	6	14	20
14	98	7	13	20
13	91	7	12	19
12	96	8	11	19
11	99	9	10	19
10	100	10	9	19
9	99	11	8	19
8	96	12	7	19
7	98	14	6	20

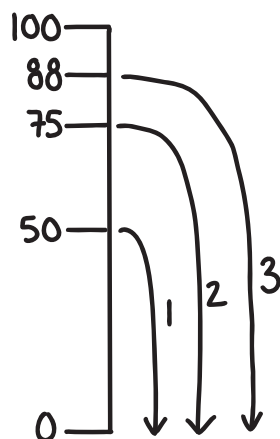
It seems like 19 is the best worst-case scenario. But what if we changed the way we thought and decreased the interval as we went. For example, if we want to try to create a worst-case scenario of 18, we would drop the first glass from floor 18. If it breaks, then we have to use the second glass to test up to 17 more floors (18 drops in total). But, what if the first glass survives the drop from floor 18? Well, then we drop it from floor 35 (18 + 17) next. If it breaks, we have to now test 16 floors between floors 18 and 35 (18 drops in total). If the first glass survives floor 35, the next drop is floor 51 (18 + 17 + 16) and so on. The worst-case scenario on this is if the first glass breaks on floor 18, 35, 51, 66, 80, or 93, all of which will require 18 drops in total. We can use the same thinking to force worst-case scenarios of 17, 16, 15.

18	17	16	15
18 ↓ +17 35 ↓ +16 51 ↓ +15 66 ↓ +14 80 ↓ +13 93	17 ↓ +16 33 ↓ +15 48 ↓ +14 62 ↓ +13 75 ↓ +12 87 ↓ +11 98	16 ↓ +15 31 ↓ +14 45 ↓ +13 58 ↓ +12 70 ↓ +11 81 ↓ +10 91 ↓ +9 100	15 ↓ +14 29 ↓ +13 42 ↓ +12 54 ↓ +11 65 ↓ +10 75 ↓ +9 84 ↓ +8 92 ↓ +7 99

How low can this go? For example, can we force a worst-case scenario of 10? No. We can't do this because the first drop would need to be at floor 10, then 19, and so on. $10 + 9 + 8 + \dots + 1 = 55$. So, we are looking for $1 + 2 + 3 + \dots + N \geq 100$. We have seen that $N = 18, 17, 16$, and 15 work. So does $N = 14$. But $N = 13$ does not ($1 + 2 + 3 + \dots + 13 = 91$). So, 14 drops is the best worst-case scenario we can find for this task.

STUDENT SOLUTIONS

A.



Commentary:

This group has started doing a sort of binary search—repeatedly splitting the building in half. It is clear that they are building their strategy on the assumption that the first glass doesn't break until they are ready for it to.

Suggested hints:

What is the worst thing that can happen with this strategy?

So, what happens if the glass breaks on the first drop?

(Continued)

(Continued)

B.

2
4
6 > 7
8 > 9
10 > 11
12
⋮
⋮
100 > 99

50+1

Commentary:

This is a common strategy—dropping from every second floor. This strategy minimizes the number of drops that you need to make with the second glass, once the first one breaks. But, there could be a lot of drops for the first glass.

Suggested hints:

What is the worst-case scenario for this strategy?

What if the first glass doesn't break until floor 100?

So, you are trying intervals of 2. How many drops would it be if the interval was 3?

C.

100
90
80
70
60
50
40 (10)
30
20
10

99
98
97
96 (9)
95
94
93
92
91

Commentary:

This group has chosen intervals of 10. And, to be sure they do not make any mistakes, have listed every number between 90 and 100. Often groups who do an interval of 10 assume that this 10 is the ideal interval.

Suggested hints:

What if you tried intervals of 11?

What if you tried intervals of 9?

How many different intervals will have a worst-case scenario of 19?

D.

100 } 10
90 }
75
60
45
30
15

17

Commentary:

When not using an interval of 10 or 20 (or an interval that divides into 100), there is an assumption that the worst-case scenario occurs when the first glass breaks on floor 100. This is not true. In this, the worst-case scenario occurs when the glass break at floor 90. This group has also made the mistake of assuming that there are 10 floors between floors 90 and 100.

Suggested hints:

How many floors are between floor 90 and 100?

What if the first glass broke on floor 75? How many total drops in the worst-case scenario?

Is there a worse-case scenario?

E.	Commentary:	Suggested hints:
55	This group is exploring a decreasing interval. Their thinking is bang on for a shorter building.	What is the worst-case scenario if the first glass broke on floor 10, 19, 27, 52, 54, and 55? Maybe try to create a large worst-case scenario. What if the first drop was 15 floors?
54		
52		
49		
45		
40		
34		
27		
19		
10		

F.	Commentary:	Suggested hints:
100 93 80 66 51 35 18 $7 + 6 = 13$	Like example D, this group has assumed that the worst-case scenario occurs if the first glass breaks on floor 100. Otherwise, they are doing a great job with their decreasing interval.	What would happen if the first glass broke when dropped from floor 93? Is 18 the best first interval?

CONSOLIDATION

When consolidating this lesson, it is important to focus on the idea of the best worst-case scenario. This can be done by looking at any strategies on the whiteboards and talking about the worst thing that can happen for that strategy. Keep referring back to this as you look at different intervals that groups tried.

CHECK-YOUR-UNDERSTANDING QUESTIONS

MILD

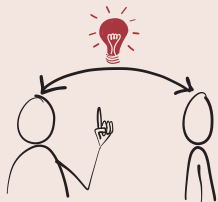
- For a 50-floor building, what is the worst-case scenario if the strategy is to drop the first glass off floors 10, 20, 30, 40, and 50?
- For a 200-floor building, what is the worst-case scenario if the strategy is to drop the first glass off floors 25, 50, 75, . . . , 200?

MEDIUM

- For a 50-floor building, what is the best worst-case scenario?
- For a 200-floor building, what is the best worst-case scenario?

SPICY

- A. For a 100-floor building one group has decided to drop from floors 20, 38, 54, What is the worst-case scenario?
- B. For a 1000-floor building what is the best worst-case scenario?



Author Notes

One of the things that will happen in this lesson is that students will use subtraction to calculate how many floors there are between two specific floors. This doesn't work. For example, between floor 10 and 20, there are 9 floors—not 10. Be prepared to ask students to write out each of the floors.

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Notes to My Future Forgetful Self

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